second), $(p-1)$ may be filled by 1's in ${}_rC_{p-1}$ ways, and each of these different ways gives rise to $9^{r-(p-1)}$ integers containing the digit 1 p times. Hence, in the first subinterval the number of integers containing 1 at least p times is equal to the sum of expression (1) and ${}_rC_{p-1}\cdot 9^{r-p+1}$. And, therefore, the number of integers in the interval $10^r \leqq x < 10^{r+1}$ which do not contain the digit 1 at least p times, $p \leqq r$, is

$$9\cdot[\text{first } p \text{ terms of expansion of } (9+1)^r] - {}_rC_{p-1}\cdot 9^{r-p+1},$$

or

$$9\cdot\{[\text{first } p \text{ terms of expansion of } (9+1)^r] - {}_rC_{p-1}\cdot 9^{r-p}\}.$$

253. Proposed by HERBERT N. CARLETON, West Newbury, Massachusetts.

Prove that $n^{2k+8} - n^{2k} \equiv 0 \pmod{20}$ for integral values of n and k.

Solution by R. M. Mathews, Riverside, California.

$$n^{2k+8} - n^{2k} = n^{2k}(n^2-1)(n^2+1)(n^4+1)$$

When n is even, $n^{2k} \equiv 0 \pmod 4$. When n is odd, $n^2 - 1 \equiv 0 \pmod 4$.

Next, n being an integer must be of the form $5m$, $5m \pm 1$, or $5m \pm 2$.

For n of the form $5m$, $n^{2k} \equiv 0 \pmod 5$; for n of the form $5m \pm 1$, $n^2 - 1 \equiv 0 \pmod 5$; and for n of the form $5m \pm 2$, $n^2 + 1 \equiv 0 \pmod 5$.

Hence, $n^{2k+8} - n^{2k} \equiv 0 \pmod{20}$, n and k being integers. This is also true of $n^{2k+4} - n^{2k}$.

Also solved by O. S. Adams, W. J. Thome, Elijah Swift, E. B. Escott, C. C. Yen, and the Proposer.

QUESTIONS AND DISCUSSIONS.

Send all communications to U. G. Mitchell, University of Kansas, Lawrence.

REPLIES.

34. Given the mixed integral and functional equation

$$\int_{x=0}^{x=x} f(x)dx = \frac{h}{6}\left[f(0) + 4f\left(\frac{x}{2}\right) + f(x)\right],$$

to determine the function $f(x)$. This equation is of rather fundamental practical value as it has to do with the most general solid whose volume is given by the prismatoid formula.

Remark by S. A. Corey, Albia, Iowa.

The prismoidal formula gives the exact value of this integral whenever the fourth derivative of $f(x) = 0$. This was shown in an article entitled "Certain Integration Formulæ Useful in Numerical Computation" in Vol. XIX, Nos. 6 and 7, of this Monthly, in which formula $(1r)$ is the prismoidal formula including an expression for the remainder term.

$f(x) = Ax^3 + Bx^2 + Cx + D$ is, therefore, the most general value of the function $f(x)$ for which the prismoidal formula gives the exact value for all values of x.

28. Is it possible to obtain $\int \cos\theta^2 d\theta$ without expanding $\cos\theta^2$? If it is not, can some interesting properties of this integral be determined by treating it as a special function?